

Fourier Transform

* Represent (transform) a non-Periodic Function from time domain to Frequency domain

$$T.D \xrightarrow{F.T} F.D$$

$$g(t) \xrightarrow{F.T} G(f)$$

$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi f t} \cdot dt$$

I.F.T.

$$F.D \xrightarrow{I.F.T} T.D$$

$$G(f) \xrightarrow{I.F.T} g(t)$$

$$g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{+j2\pi f t} \cdot df$$

① Find F.T of the rectangular pulse shown.

$$g(t) = A \cdot \text{rect}\left(\frac{t}{T}\right) \\ = A \cdot \text{rect}\left(\frac{t}{T}\right)$$

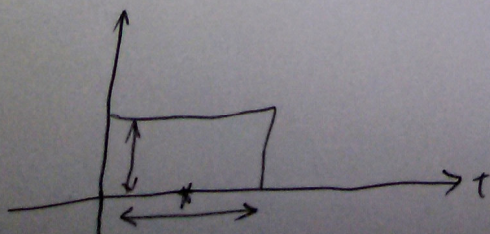
to draw rect

1- amplitude (A)

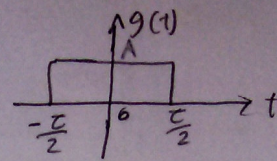
2- Center

نصف العرض في المبدأ
والنصف في المبدأ

3- Width



$$A \cdot \text{rect}\left(\frac{t}{T}\right)$$



$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi f t} \cdot dt$$

$$= \int_{-T/2}^{T/2} A \cdot e^{-j2\pi f t} \cdot dt$$

$$= \frac{A}{-j2\pi f} \left[e^{-j2\pi f t} \right]_{-T/2}^{T/2}$$

$$= \frac{A}{-j2\pi f} \left[e^{-j2\pi f \frac{T}{2}} - e^{+j2\pi f \frac{T}{2}} \right]$$

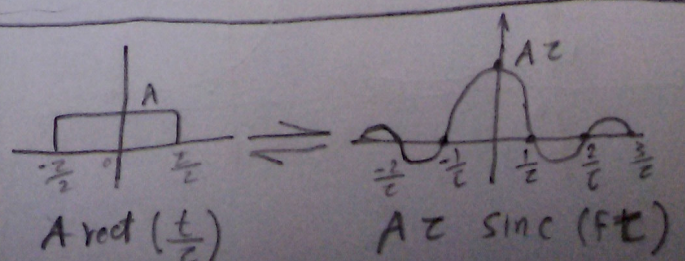
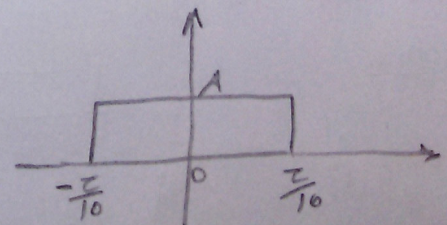
$$= \frac{A}{\pi f} \left[\frac{e^{+j\pi f T} - e^{-j\pi f T}}{2j} \right]$$

$$= \frac{A T}{\pi f T} \sin(\pi f T)$$

$$= A T \cdot \text{sinc}(f T)$$

$$A \cdot \text{rect}\left(\frac{t}{T}\right) \iff A T \text{sinc}[f T]$$

$$A \cdot \text{rect}\left(\frac{3t}{T}\right) = A \cdot \text{rect}\left[\frac{t}{T/3}\right]$$

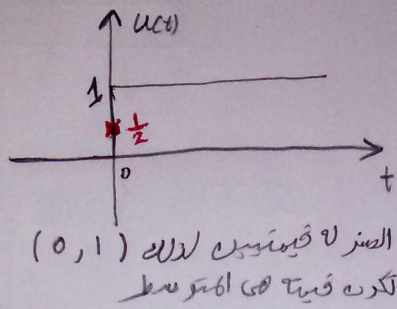


* Some Important Functions:-

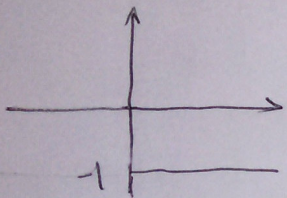
① Unit Step Function $u(t)$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

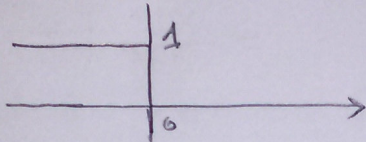
$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases}$$



$$-u(t)$$

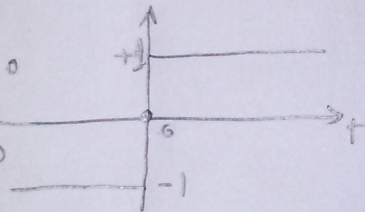


$$u(-t)$$



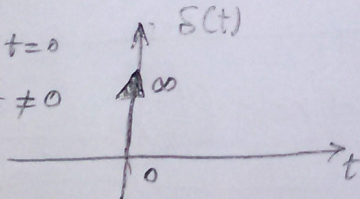
② Signum Function $\text{sgn}(t)$

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$



③ Delta Function $\delta(t)$

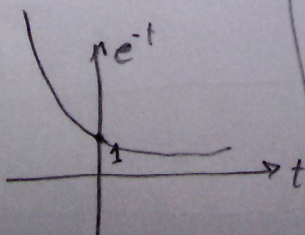
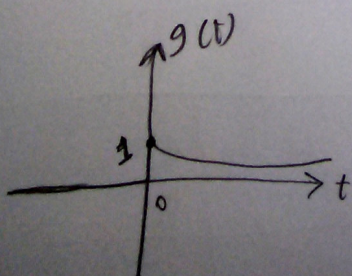
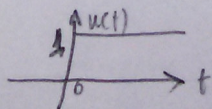
$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$



$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

② Find F.T. For $g(t) = u(t) \cdot e^{-t}$

* Draw



$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi ft} dt$$

$$= \int_0^{\infty} e^{-t} \cdot e^{-j2\pi ft} dt$$

$$= \frac{1}{-j2\pi f - 1} \left[e^{-t(j2\pi f + 1)} \right]_0^{\infty}$$

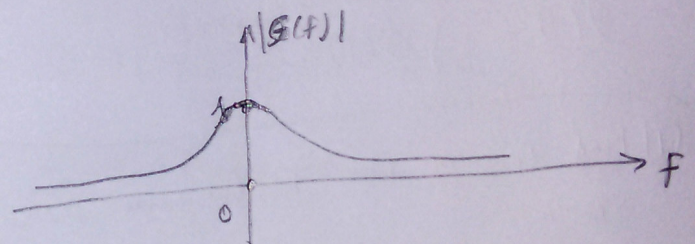
$$= \frac{1}{-(1 + j2\pi f)} \left[e^{-\infty} - e^0 \right]$$

$$= \frac{1}{1 + j2\pi f}$$

$$u(t) \cdot e^{-t} \longleftrightarrow \frac{1}{1 + j2\pi f}$$

* Draw in F.D

$$|G(f)| = \frac{1}{\sqrt{1 + 4\pi^2 f^2}}$$



2

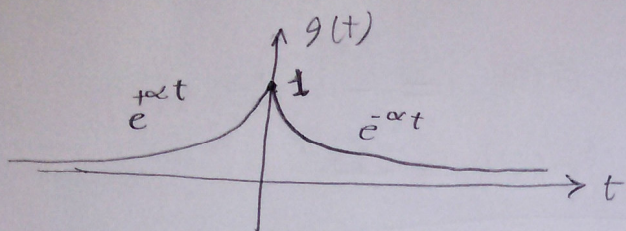
Sec 3

إتصالات

*Ex: Find F.T. of $g(t) = e^{-\alpha|t|}$

$$|t| = \begin{cases} t & t \geq 0 \\ -t & t < 0 \end{cases}$$

$$g(t) = \begin{cases} e^{-\alpha t} & t \geq 0 \\ e^{\alpha t} & t < 0 \end{cases}$$



$$\begin{aligned} G(f) &= \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi f t} dt \\ &= \int_{-\infty}^0 e^{\alpha t} \cdot e^{-j2\pi f t} dt + \int_0^{\infty} e^{-\alpha t} \cdot e^{-j2\pi f t} dt \\ &= \left. \frac{e^{t(\alpha - j2\pi f)}}{\alpha - j2\pi f} \right|_{-\infty}^0 + \left. \frac{e^{-t(\alpha + j2\pi f)}}{-(\alpha + j2\pi f)} \right|_0^{\infty} \\ &= \frac{e^0 - e^{-\infty}}{\alpha - j2\pi f} - \frac{e^{-\infty} - e^0}{\alpha + j2\pi f} \\ &= \frac{1}{\alpha - j2\pi f} + \frac{1}{\alpha + j2\pi f} \\ &= \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2} \end{aligned}$$

$$e^{-\alpha|t|} \iff \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$$

* Properties of Fourier Transform

F.T. مجموعة من الخواص تساعدنا في إيجاد F.T. لموال مجهولة بمعلومية وال أخرى وليس عن طريق التكامل

1 Linearity (Superposition)

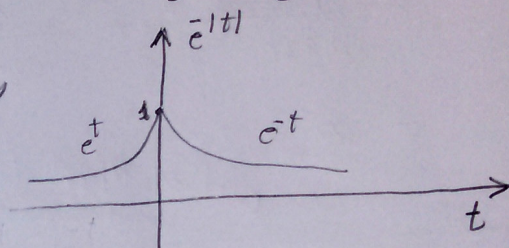
$$\text{Let } g_1(t) \iff G_1(f)$$

$$g_2(t) \iff G_2(f)$$

$$a g_1(t) \pm b g_2(t) \iff a G_1(f) \pm b G_2(f)$$

Ex: Find F.T. for $g(t) = e^{-|t|}$

Using Linearity



$$g(t) = e^{+t} u(-t) + e^{-t} u(t)$$

$$\therefore e^{-t} \cdot u(t) \iff \frac{1}{1 + j2\pi f}$$

$$\therefore e^{+t} \cdot u(-t) \iff \frac{1}{1 - j2\pi f}$$

$$\therefore G(f) = \frac{1}{1 + j2\pi f} + \frac{1}{1 - j2\pi f}$$

2 Time Scaling

$$g(t) \iff G(f)$$

$$g(at) \iff \frac{1}{|a|} G\left(\frac{f}{a}\right)$$

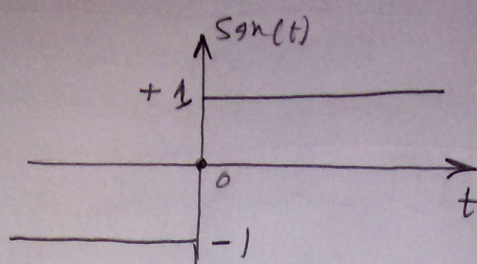
Ex: $g(t) = e^{-\alpha t} \cdot u(t)$ $u(at) = u(t)$

$$\therefore e^{-t} \cdot u(t) \iff \frac{1}{1 + j2\pi f}$$

Using Time Scaling

$$\therefore g(t) = e^{-\alpha t} u(at) \iff \frac{1}{|a|} \cdot \frac{1}{1 + j2\pi \frac{f}{a}}$$

Ex: $g(t) = \text{sgn}(t)$



$$G(f) = \int_{-\infty}^0 -1 \cdot e^{-j2\pi ft} dt + \int_0^{\infty} 1 \cdot e^{-j2\pi ft} dt$$

$$= - \left. \frac{e^{-j2\pi ft}}{-j2\pi f} \right|_{-\infty}^0 + \left. \frac{e^{-j2\pi ft}}{-j2\pi f} \right|_0^{\infty}$$

$$= \frac{[e^0 - e^{\infty}]}{j2\pi f} + \frac{e^{-\infty} - e^0}{-j2\pi f}$$

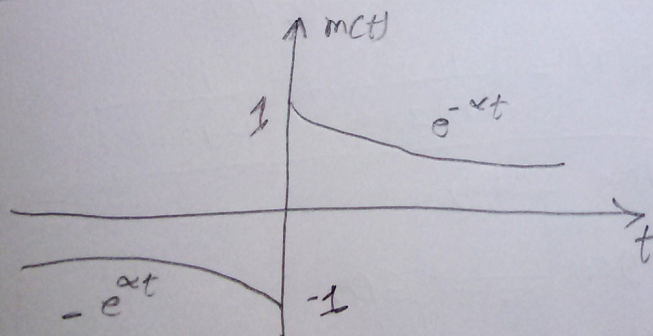
$= \infty$

do kaly vishal da chiz

$m(t)$

$\text{sgn}(t) = \lim_{\alpha \rightarrow 0} [m(t)]$

F.T. $[\text{sgn}(t)] = \lim_{\alpha \rightarrow 0} [M(f)]$



$\text{sgn}(t) = \lim_{\alpha \rightarrow 0} m(t)$

$m(t) = -e^{-\alpha t} \cdot u(-t) + e^{-\alpha t} u(t)$

$\therefore e^{-\alpha t} \cdot u(t) \Rightarrow \frac{1}{\alpha + j2\pi f}$

$\therefore -e^{-\alpha t} \cdot u(-t) \Rightarrow -\frac{1}{\alpha - j2\pi f}$

using super position

$M(f) = \frac{1}{\alpha + j2\pi f} - \frac{1}{\alpha - j2\pi f}$

$= \frac{-j4\pi f}{\alpha^2 + 4\pi^2 f^2}$

F.T. $[\text{sgn}(t)] = \lim_{\alpha \rightarrow 0} \frac{-j4\pi f}{\alpha^2 + 4\pi^2 f^2}$

$G(f) = -j \frac{1}{\pi f}$

$|G(f)| = \frac{1}{\pi f}$

$\text{sgn}(t) \rightleftharpoons \frac{1}{j\pi f}$

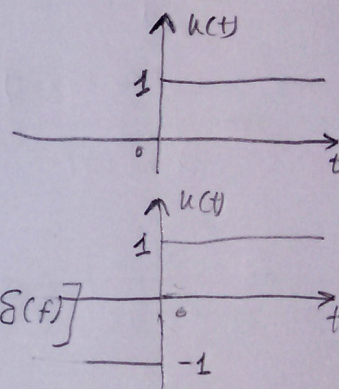
Ex: $g(t) = u(t)$

$\text{sgn} \rightleftharpoons \frac{1}{j\pi f}$

$u(t) = \frac{1}{2} [\text{sgn}(t) + 1]$

using super position

$u(f) \rightleftharpoons \frac{1}{2} \left[\frac{1}{j\pi f} + \frac{1}{2} \delta(f) \right]$



$a \rightleftharpoons a \delta(f)$